Possible Use of Already Formalized Mathematical Knowledge

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1 Motivation

In this sessions the problems with a fixed representation language for QED have been discussed. In particular the objections of Gérard Huet against the feasibility of QED presented in the QED discussion at CADE-12 (1994) were taken as starting point. Huet said that there will be never any consent on the logical framework (type theory, set theory, constructive logic, classical logic, generic logic, ...). If this is true, QED must provide different formal systems for the representation of mathematical knowledge.

It seems to be quite obvious that different formalizations have their own advantages and drawbacks. Even for one single problem it might be that different formulations are appropriate, for instance, an explicit one in which a user can easily represent and recognize a theorem, and a more implicit one which is more suitable for a fully mechanical or a machine-supported proof. The advantages of different representations can easily be seen by means of sorts. In many situations a sorted formulation is more adequate for the formalization and the proof process (automatic as well as interactive) than an unsorted one. In some cases, however, it is not possible to use a (standard) sorted formalization, for instance, when you want to make a statement that a certain term has *not* a particular sort. The same situation holds for type theory compared to set theory.

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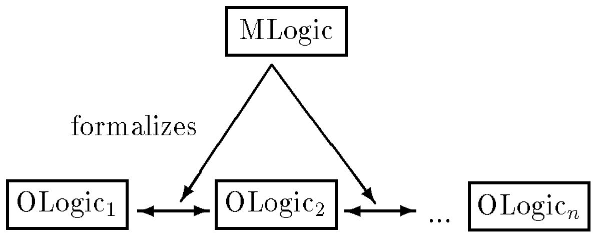
Another important aspect concerns the reuse of big amounts of mathematical knowledge that has been accumulated in systems like Mizar or Nuprl and should be reusable in QED.

2 Approach

As a consequence of the problems with one fixed language, QED should not only support one single formal language, in which all statements have to be made, but a (not too big) variety of languages including the standard approaches (like set theory or type theory). In order to be able to transfer proofs from one format to another, it is necessary to have an exchange format for different syntactic objects, namely terms, formulae, assertions (axioms, definitions, theorems with their proof status), and proofs. This exchange format should satisfy in particular the requirements that it is easy to parse (prefix notation). Therefore it should be different from a user-friendly high-level format that can easily be read by humans.

In order to avoid a variety of unrelated languages, they must have provable relationships. Ideally there is one *constructive meta-logic*, for instance, the Nuprl-logic[1], in which the relationships between different formalizations can be formally proved. The main advantage of a constructive meta-logic consists in the possibility to extract from each meta-level proof an algorithm, which translates proof from one system to another.



While a classical meta-language can be used for the purpose of stating the relationship between different object logics as well, we propose to use a *constructive* meta-language. This has the advantage that by the constructive meta-proofs idealists can directly employ the theorems of another theory just by translating the theorem in their own language (and thereby

[1]see: Robert L. Constable et al., *Implementing Mathematics with the Nuprl Proof Development System.* Prentice Hall, 1986.

avoiding the reconstruction of possibly lengthy proofs), while nominalists[2] can translate the theorems *and* the proofs in their own formalism. The proposed framework allows easily to integrate existing systems in the QED system.

3 Discussion

In the discussion the following points were made:

- There should be one common meta-logic only. Is PRA the best choice for a meta-logic?
- After the discussion of the proposal that there should be only one single object logic, the idea arose that there could be a tower of object languages such that each can be encoded in a lower level one, with some set theory as the least level one.
- In oder to restrict the number of object logics to a small number certain types of languages have to be offered like set theory and type theory.
- The meta-logic should include a theory of definitions and expansion of definitions.
- An alternative to a meta-logic would be the approach of having a collection of (program-level) translations between different formalizations and proof checking on this level.
- As the last point the question was discussed on which level proofs should be communicated. The general opinion was that as much information as possible should be contained in the proofs.

[2] For the notions idealist and nominalist in the context of automated theorem proving see: Francis Jeffry Pelletier. The philosophy of automated theorem proving. In John Mylopoulos and Ray Reiter, editors, *Proceedings of the 12th IJCAI*, pages 538–543, Sydney, 1991. Morgan Kaufmann.