The organization of a data base of mathematical knowledge

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The talk started with a survey of the main difficulties the QED project has to face. These are, among others:

- Different foundational formalisms
- Different concepts of validity of theorems
- Consequently, different calculi yielding different proofs

Two different approaches can be envisaged for solving these difficulties. On the one hand, one can define a "root logic" serving as a meta-language for expressing and communicating statements and proofs developed in the particular object logics. Such an approach was judged to be a rather far-reaching goal not attainable in the near future.

On the other hand, a database of formalized mathematics and computer science can be constructed, starting from the large corpus of theories developed for some of the current systems. Apart from demonstrating the utility and applicability of formalized mathematics to "the public", such a database could improve the exchange of results within the QED community. For meeting this goal, the database would have to include a detailed description of objects, such as:

- Terminology, definitions
- Axiomatizations
- Mathematical theories
- Theorems

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- Proofs and proof methods / tactics

A theory could for example be indexed by the following information:

- On which other theories does it depend?
- Which theories does it extend?
- Are there theory interpretations to other theories?
- Over which theories is it parameterized?

Apart from that, some meta-information would have to be added in order to account for different philosophies of mathematics. Such information might detail whether a proof uses a constructive or non-constructive argument or whether it is based on a contested axiom.

The discussion following the talk centered on the problem of retrieving information from a database. Since most information is currently stored in the form of text files, it is difficult to organize the data in such a way that they are amenable to "semantic" queries as opposed to a purely textual search. In particular, experience shows that finding previously proved theorems is a serious problem. It seems that so far no adequate solutions for these problems are known and further research on this topic is necessary.

Another issue addressed during this discussion (and resumed in several other occasions) was the question of whether QED should get involved in foundational debates, for example when deciding on the structure and contents of a database, or whether QED had not better develop practical methods for making existing formalizations accessible and for combining them. This could eventually lead to a methodology of problem solving similar to the approach advocated in software engineering: Decomposing a given problem into subproblems, until solutions can be found by the aid of powerful libraries.

One of the tangible results of the discussion was the proposal to provide easy access to the systems developed in the QED context via WWW. Links to systems could be kept on a centralized WWW page. By activating a link, the user could start a session with the respective system and either initiate proof tasks or at least submit queries about certain theories. It was mentioned that some systems (Nuprl, IMPS) already provide some of these services or will soon make them available.