The Mutilated Chessboard Problem - *checked by Mizar*

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Abstract

The problem presented by John McCarthy during his lecture "Heavy duty set theory"[1] has been resolved here. The final version of that article has been made with the commitment of Andrzej Trybulec.

Proofs have been written in Mizar language and were checked by PC Mizar system. In this paper only definitions and theorems are published apart from proofs (this is the so called 'mizar abstract'). Then it has been automatically translated into English. The format of this article is exactly the same as in journal *Formalized Mathematics*.

MML Identifier: `M_BOARD`.

The articles [10], [12], [2], [13], [14], [5], [4], [6], [11], [9], [8], [7], [3], and [1] provide the notation and terminology for this paper.

We follow a convention: x, z will be sets, i, j, k will be natural numbers, and u, v will be elements of $[:\mathbb{N}, \mathbb{N}:]$.

One can prove the following proposition

(1) If $(i \bmod k) + (j \bmod k) < k$, then $(i+j) \bmod k = (i \bmod k) + (j \bmod k)$.

Set $Board = [:\mathrm{Seg}\,8, \mathrm{Seg}\,8:]$ and $MBoard = Board \setminus \{[1,1],[8,8]\}$.

Let x be an element of $[:\mathbb{N}, \mathbb{N}:]$. Then $x_{\mathbf{1}}$ and $x_{\mathbf{2}}$ are natural numbers.

Let x_1, x_2 be elements of $[:\mathbb{N}, \mathbb{N}:]$. We say that x_1 is adjacent to x_2 if and only if:

(Def.1) $|(x_1)_{\mathbf{1}} - (x_2)_{\mathbf{1}}| = 1$ and $(x_1)_{\mathbf{2}} = (x_2)_{\mathbf{2}}$ or $|(x_1)_{\mathbf{2}} - (x_2)_{\mathbf{2}}| = 1$ and $(x_1)_{\mathbf{1}} = (x_2)_{\mathbf{1}}$.

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[1]In this Report see: *The Mutilated Checkerboard in Set Theory* by John McCarthy.

Let x be a set. We say that x is domino-on-board if and only if:

(Def.2) $x \subseteq \mathit{Board}$ and $\overline{\overline{x}} = 2$ and for all elements x_1, x_2 of $[: \mathbf{N}, \mathbf{N} :]$ such that $x_1 \neq x_2$ and $x_1 \in x$ and $x_2 \in x$ holds x_1 is adjacent to x_2.

Let z be a set. We say that z is partial-covering if and only if the conditions (Def.3) are satisfied.

(Def.3)(i) For every set x such that $x \in z$ holds x is domino-on-board, and

(ii) for all sets x, y such that $x \in z$ and $y \in z$ holds $x = y$ or $x \cap y = \emptyset$.

Let z be an element of $[: \mathbf{N}, \mathbf{N} :]$. The functor $\mathrm{color}(z)$ yielding a natural number is defined by:

(Def.4) $\mathrm{color}(z) = (z_{\mathbf{1}} + z_{\mathbf{2}}) \bmod 2$.

The following three propositions are true:

(2) Let z be a set. Suppose z is partial-covering. Let x be a set. Suppose $x \in z$. Then there exist elements x_1, x_2 of $[: \mathbf{N}, \mathbf{N} :]$ such that $x = \{x_1, x_2\}$ and $\mathrm{color}(x_1) = 0$ and $\mathrm{color}(x_2) = 1$.

(3) For every set z such that z is partial-covering holds $\{u : u \in \bigcup z \ \wedge\ \mathrm{color}(u) = 0\} \approx \{v : v \in \bigcup z \ \wedge\ \mathrm{color}(v) = 1\}$.

(4) It is not true that there exists a set z such that z is partial-covering and $\bigcup z = \mathit{MBoard}$.

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