EMS
European Mathematical Society
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Committee on Mathematics Education

Niveaux de référence pour l'enseignement des mathématiques en Europe
Reference levels in School Mathematics Education in Europe

National Presentation

RUSSIA

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Mars 2000

1. General description of the mathematics teaching context

1.1. Description of the country school system. For the past few years, the school system in Russia has been in the state of permanent reorganization; therefore, when talking of its principal features, we must remember that, presently, some of them are changing, some are old but not legislative any longer, and some are new, but so far not supported by legislation. As a bright (though not most fundamental) example of this state of affairs we mention the fact that general education in Russia is actually a 10-year one, but the grades are enumerated from 1 to 11 (number 4 is usually omitted). The reason for this is that passage to the general 11-year education has been planned but not fully effected.

The Russian general education splits into 3 stages: primary school (from grade 1 to grade 3; starts at the age of 7), basic school (from grade 5 to grade 9; a student usually ends basic school at 15 years old), and senior school (grades 10 and 11, up to 17 years old). Only basic education (9 grades) is officially compulsory; however, at present the overwhelming majority of students continue education after grade 9 (in some form). Among such forms, two are the main ones: senior school itself, and vocational school. It is worth mentioning that in the framework of the latter most general disciplines are taught in accordance with a curriculum close to that in the general school. As a rule, the splitting into 3 stages indicated above does not require that a student change the educational establishment where he or she learned at an earlier stage. Most frequently,

a general school embraces all three stages, and many students go to one and the same school from grade 1 to grade 11.

The general education system in Russia has always been rigidly centralized. For many years the entire country has had not only the same curriculum, but also the same text-books. Moreover, a trend was clearly seen to the unification of the methods of teaching. It is not until the past few years that this situation started to change; at present different text-books are used in parallel, as well as different curricula, and part of decisions are taken at the regional level and even at the school level. For instance, the school authorities have some freedom in allocating the number of hours per week to every discipline in every grade, however within the bounds prescribed at the national and the regional level. All this has made realization of multy-level education possible; the process of passage to multy-level teaching is now in progress for almost all school subjects, not only for mathematics and physics where the traditions of 2- or 3-level teaching are older.

In most cases, assessment of the results of teaching is made by a school and a teacher. However, there exist all-nation unified exams after grade 9 and after grade 11. The texts of the written exams are prepared by the Ministry of Education (or are approved by that Ministry in case of experiments). Exemplary lists of questions for the oral exams are also issued by the Ministry. At present, as long as no all-national standards or minimal requirements on the results of education are enacted, the texts of exams serve as a key tool for unification of the results of learning.

Also, attempts are made towards introducing a unified all-nation entrance exam for those who wish to continue education at a university. First experiments in this direction will be carried out in several regions of Russia in the course of the university-entrance campaign of 2001. This exam is planned to have a multi-choice test form; once implemented, it will, surely, have great impact on the unification of what is taught in mathematical classrooms.

1.2. Place and importance of mathematics in the curriculum. Russian traditions put mathematics at one of the leading places in

education. Mathematics is viewed as necessary not only because it provides backgrounds for other disciplines or means for solving «real life» problems, but also because it contributes most to personal development of a student. As a school subject, mathematics is an inherent part of any curriculum in any grade. A particular role of mathematics is emphasized by the fact that an exam in mathematics is compulsory for all students graduating from grade 9 and from grade 11, independently of the type and orientation of a school. Also, such a leading role manifests itself in a relatively large number of class hours allocated to mathematics, and in the popularity of the schools and classes with a deeper study of mathematics. It should be noted that the process of differentiation undergone by the Russian school system has already resulted not only in an abrupt increasing of the number of special mathematical schools, but also, on the contrary, in the emerging of schools and classes where the teaching of mathematics is reduced (up to 3 hours per week in senior school). In such classes, general development of a student is viewed as a first-rate problem, and specific skills are taught in less detail.

Finally, as an important symptom indicating that the importance of studying mathematics is broadly acknowledged, we mention that the learning of not obligatory, optional mathematics is widely spread; such studies may have various, mostly out-of-class, forms, including many kinds of mathematical competitions.

Traditionally, in grades 7–9 mathematics comprises 2 school subjects, ***geometry*** and ***algebra*** that are taught in parallel (e.g., 2 hours per week for geometry, and 3 hours for algebra). In the senior school, also two subjects are taught, ***geometry*** and ***algebra and elements of calculus***. In grades 5 and 6 (and in primary school) we have a united course of ***mathematics***.

The mathematics curricula prescribe a rather high level of treatment of theoretical concepts, as well as achievement of relatively strong computational skills. Mathematics (especially geometry) is regarded as a strict, deductively constructed branch of science. On the other hand, the Russian traditions of teaching mathematics are to a certain extent archaic: except in the special mathematical schools, no elements of probability

theory, statistics, combinatorics, or data processing are present in classroom mathematics in Russia.

However, this state-of-affairs begins to change: some of new textbooks for basic and senior school include chapters devoted to there topics, which, nevertheless, remain optional.

2. Main mathematics objectives

The national mathematics curriculum indicates the following objectives of learning mathematics at school:

- mastering specific mathematical skills necessary for applications to practice, for the study of other disciplines, and for continuation of education;
- developing the students intellectually, forming the features of the mathematical way of thinking necessary for productive participation in the life of the modern society;
- forming the view on mathematics as a source of tools for describing and studying the real world;
- forming the view on mathematics as part of all-human culture.

This list is in good agreement with the structure proposed. Moreover, the last of these objectives suggests that the students realize the general-culture value of mathematics.

However, the traditional technocratic trends in the understanding of the role and place of mathematics in school mathematics in Soviet Russia have led to distortions: the main efforts are directed towards achievement of the first objective among those indicated above.

As to the other three objectives, they remain rather declarative, being achieved to the extent at which this is linked with acquiring specific knowledge. Seldom can we see application of methods of teaching especially designed in correspondence with these objectives, nor do they find reflection in the descriptions of the content of school education.

It is convenient to discuss the structure of the main mathematics objectives in more detail in the next section, in close relationship with the content of the course.

3. Basic content

Despite the customary high-level unification of the school system in Russia, the past decade gave birth to a great variety of versions of teaching school mathematics. Since, largely, these versions differ from one another in the amount of the material studied and in the general orientation of a given school (the «profile» of the school), for the description of the possible versions of the content of school mathematics we can evoke a two-dimensional scheme in which any educational type is characterized by two parameters, the ***profile*** and the ***level***.

We distinguish 3 profiles: arts (A), general (G), and specialized mathematical (M), and 3 levels: minimal (1), basic (2), and deepened (3). Thus, G_2 correspond to the version of the general (non-specialized) education at the basic level, and M_3 symbolizes the specialized and deepened study of mathematics. Since the extreme cases M_1 and A_3 should, seemingly, be excluded from consideration, we arrive at the following pattern of 7 possible versions:

	M_2	M_3
G_1	G_2	G_3
A_1	A_2	

All these versions are realized in some schools; it is worth noting that the teaching of mathematics in the schools of A-profile differs greatly in its aims from the traditional teaching. The goals corresponding to the A-versions, as well as the problem of creation of adequate curricula and methods of teaching, are as yet far from being articulate. Generally, the teaching materials especially elaborated for the A-versions are more focused on interpretation of mathematical facts and pay less attention to technical skills.

In the federal curricula, the variety of versions described above is supported as follows. There is a basic curriculum recommended by the Ministry of Education ($\approx G_2$), and its description includes the list of obligatory

minimal requirements on the content of education and on the knowledge of mathematics after graduation (≈G_1). This obligatory minimal content constitutes the common core of all versions. Besides, an especial curriculum was enacted at the federal level for the specialized mathematical schools and classes (≈M_3).

Below we present a description of the content of the school mathematics teaching, as given in the standard federal curriculum (G_2) for the basic school; at present, most of the Russian students are taught in accordance with precisely this curriculum.

Grades 5 to 9

Numbers and computations

Positive integers and the zero. Decimal scale. Operations with natural numbers. Properties of operations. Powers with natural exponent.

Divisors and multiplies. Criteria for divisibility. Primes. Representation of a number as a product of primes.

Fractions. The main property of the fractions and its applications to transformations of fractions. Comparison of fractions. Operations with fractions. Finding a part of a number and recovering a number by its part.

Decimal fractions. Comparison of decimal fractions. Operations with decimal fractions. Representation of ordinary fractions by decimal ones. Arithmetic mean.

Ratios, proportions. Direct and inverse proportionality. Percentages. Basic problems on percentages. Solution of world problems by arithmetical means. Positive and negative numbers. Opposite numbers.

Absolute value of a number. Comparison of positive and negative numbers. Operations with positive and negative numbers, properties of operations.

Integers. Rational numbers. Correspondence between numbers and points on a coordinate line. The concept of an irrational number.

Number inequalities and their properties. Termwise addition and multiplication of inequalities.

Approximations. Absolute and relative errors. Rounding off natural numbers and decimal fractions. Preliminary estimation of the results of computations.

Square root. Decimal approximations to the square root of a number.

Computations with the help of a calculator.

Expressions and transformation

Expressions with letters. Number substitutions in expressions with letters. Calculations by a formula. Letter form of the properties of arithmetical operations.

Powers with integral exponent and their properties. Operating with parentheses, collecting similar terms. Addition and multiplication of polynomials. Formulas for $(\boldsymbol{a}+\boldsymbol{b})^2$, etc. Factorization of polynomials. Quadratic trinomial: singling out a complete square, factorization.

Algebraic fractions. The main properties of an algebraic fraction. Cancellation of fractions. Operations with algebraic fractions.

Properties of the arithmetic square root, with application to transformation of expressions.

Sine, cosine, and tangent of an arbitrary angle. Basic trigonometrical identities. Reduction formulas.

Arithmetic and geometric progressions. Formulas for a general term and for the sum of several terms of an arithmetic and a geometric progression.

Equations

Equations with one variable. Roots of an equation. Linear equation. Quadratic equation. Root formula for quadratic equation. The Vieta theorem. Solution of some rational equations.

Equations with two variables. Systems of equations. Solution of a linear system with two variables. Simplest nonlinear systems. Graphic

interpretation of the solutions to a system of equations with two variables. Solution of word problems with the help of equations and systems.

Linear inequalities with one variable and systems of such inequalities. Quadratic inequalities with one variable.

Functions

Cartesian coordinates on the plane. Functions. The domain and range of a function. The graph of a function. Monotone functions. Preservation of the sign on an interval.Functions: $y = kx$, $y = kx + b$, $y = \frac{k}{x}$, $y = x^3$, $y = ax^2 + bx + c$, $y = |x|$.

Tables and diagrams.

Geometric figures on the plane. Measurements.

Initial geometrical concepts. Equality of figures.

Segments. Length of a segment, distance between points.

Angles, kinds of angles. Adjacent and vertical angles. Bisectrix of an angle and its properties. Opening of an angle. Angle measures.

Intersecting and parallel lines. Orthogonal lines. Theorems on intersecting and parallel lines. Properties of the middle perpendicular to a segment. Distance from a point to a line. Distance between parallel lines.

Triangles. Elements of a triangle. Criteria for equality of triangles. The sum of the angles of a triangle. Similarity of triangles. Triangle inequality. Middle line of a triangle and its properties. Properties of isosceles and equilateral triangles. Right triangles.

The Pythagorean theorem. Sine, cosine, and tangent of an acute angle. Finding elements of right triangles. Area of a triangle.

Quadrangles. Parallelograms, their properties. Rectangles, rhombuses, squares, trapezoids. Middle line of a trapezoid. Area of quadrangles.

Polygons. Regular polygons. The sum of the angles of a convex polygon.

Circles and discs. Tangents line to a circle. Central angles and inscribed angles. The circle circumscribed about a triangle. The circle inscribed in a triangle. Length of a circle. Length of an arc of a circle. Area of a disc.

Basic geometrical constructions. Basic compasses and ruler problems.

Axial symmetry. Central symmetry.

Vectors. The angle between two vectors. Coordinates of a vector. Addition of vectors. Multiplication of a vector by a number. Scalar product of vectors.

Polyhedrons. Balls, cylinders, cones. Volume of a right parallelepiped.

It is worth mentioning that almost all students complete general education in a senior school, where they are taught elements of calculus and

3-dimensional geometry. Also, the study of mathematics in senior school involves an intensive repetition, from a new viewpoint, of the fundamental algebraic concepts occurring in basic school (number equations, inequalities, functions, etc.).

4. Exemplary topics

4.1. Quadratic equation. Quadratic equations arise in grade 8 of general school (students of 14 years old). In all versions of curricula and in all

text-books, the study of quadratic equations is prefaced by the topic «Square root», where, in particular, the students become acquainted with the standard classification of the real numbers (rational and irrational numbers). The concept of the arithmetic square root is introduced, methods of finding approximate values of the square root of a number are considered, and the basic identities for square roots are proved.

In accordance with the standard federal curriculum, the algebraic problem of solving a quadratic equation is studied first; passage to the functional viewpoint involving the properties and graphs of quadratic functions occurs later.

The obligatory content of the topic «Quadratic equations» (G_2-curriculum; see §3 above) includes the following items:

- singling out a complete square in a quadratic trinomial;
- the general formula for the roots of a quadratic equation;
- the Vieta theorem;
- the graph of a quadratic function;
- properties of quadratic functions;
- solution of quadratic inequalities with one variable.

As obligatory skills, the curriculum lists the following: ability to solve arbitrary quadratic equations and to apply such equation to solution of relevant problems, ability to construct the graphs of quadratic functions and to obtain information from graphic representations, and ability to recognize quadratic functions among other given dependencies.

In the curricula of M-type (for special schools; see §3) the content of this topic is widened substantially, especially in the part of technical skills. The students must be able to solve equations that can be reduced to a quadratic one (biquadratic, reciprocal, etc.), to determine the signs of the roots with the help of the Vieta theorem, to investigate quadratic equations and inequalities involving parameters, to analyze the data of a word problem the solution of which involves quadratic equations. Much attention is paid to systems of two quadratic equations with two variables that can be solved by reducing to a single quadratic equation.

4.2. Pythagorean theorem. The federal curriculum for general school prescribes that this theorem be treated in grade 8. The study of the proof of the theorem is viewed as compulsory for all versions of curriculum. Besides the theorem itself, the inverse theorem is also considered. Usually, the approach to the Pythagorean theorem used by the teacher in a class-room is determined by the choice of a text-book. The following approaches are adopted most frequently:

1) The Pythagorean theorem is handled after the introduction of the trigonometric functions of an acute angle in a right triangle; then,

naturally, the proof of the theorem is based on the properties of these functions.

2) The Pythagorean theorem is studied within the framework of the topic «Area of figures», which makes it possible to employ the standard proof related to arias.
3) The Pythagorean theorem is treated after the topic «Geometrical transformations» is already learnt. Then similarity serves as a main tool in the proof of the theorem.

The 3-dimensional version of the Pythagorean theorem is studied in senior school, grade 10.

4.3. Similarity. Though this topic is traditionally regarded as difficult, some facts about similarity are present in every mathematics curriculum in Russia. The standard curriculum (G_2-type) restricts the teacher's and student's attention to ***similarity of triangles***, including the Thales theorem, criteria for similarity of triangles, and applications to solving geometrical problems. The place of this topic in the course of geometry (in grade 8 or 9) is usually determined by the approach adopted in the corresponding text-book. The more general concept of similarity of arbitrary shapes and figures (or only polygons) is touched upon only in specialized schools and classes (M-type curricula); the same refers to the treatment of similarity in the framework of studying ***geometric transformations*** (central dilation, etc.). Nevertheless, the behavior of aria under similarity transformations is always discussed, but, as experience shows, poorly understood (despite the fact that proportionality and change of scales are repeatedly treated at earlier stages of teaching mathematics).

4.4. Word problems. Representing one of the core topics of school mathematics in primary and basic school, word problems serve as a principal tool for fostering the skills of operating with mathematical models, namely,

- skills of formal description of a situation given verbally;
- skills of analyzing a model;

– skills of formulating and solving the mathematical problem created on the basis of modeling.

In the school mathematics course, the main attention is paid to the last two kinds of skills; the first one requires well-developed logical thinking and general mathematical culture and, therefore, seldom arises explicitly in general school. As an exception, we mention the projects and curricula for the so-called ***developing education*** in primary school. In such curricula, the requirements concerning technical skills are weakened, but more attention is paid to the skills of analyzing a text, of describing one and the same situation in different ways, etc.

The mathematical techniques used for solving word problems complicates gradually, in accordance with the general course of studying mathematics. For instance, in grades 1 and 2 the students are given only explicitly formulated problems the solution of which requires one or two arithmetical operation (with no use of letter notation). In grades 3 and 4 the students are taught to use equations of the form $\boldsymbol{a} \pm \boldsymbol{bx} = \boldsymbol{c}$. The range of situations treated in such problems is fairly limited. The modeling of the verbal constructions «is greater by ...», «is less by ...», «is ... times greater than ...», and the like is viewed as most important. In grades 5 and 6 word problems involving percentages arise, and in grade 7 the students begin to operate with two unknowns, forming a relevant system of (linear) equations. In grade 8 word problems start to include those reducing to square equations. In senior school, in text form we find problems on maxima and minima.

Frequently, the context in which word problems are posed is related to other school disciplines (mainly, to physics and chemistry). On the other hand, the skills acquired via solving word problems within mathematics are employed in physics or chemistry classes. Word problems posed in terms of such disciplines as history or economics have come into practice only recently; largely, they occur in A-curriculum mathematics classes (see §3).

4.5. Percentages. The standard curriculum for general school specifies that percentages be taught in all types of schools and classes. As

a rule, percentages are studied at the end of grade 5 or at the beginning of grade 6. After that, percentages occur sporadically, mainly in word problems. The principal goal is to form understanding of the percentages as a specific type of indicating a portion of a quantity; it is widely agreed that the difficulties related to teaching percentages lie not in calculations but rather in understanding various verbal constructions involving percentages. We mention that, surprisingly, even now word problems on rates of interest seldom occur in school practice in Russia.

Usually, the study of percentages is closely linked with that of fractions, decimals, ratios, and proportionality. The teachers traditionally distinguish three types of problems with percentages: finding a percent of a number, recovering a number if a percent of it is known, and finding a percent ratio of two given numbers.

4.6. Identical transformations of algebraic expressions (an additional topic). Traditionally, substantial amount of time in mathematics classes in Russia is spent to create stable skills of operating with algebraic expressions. The first acquaintance with expressions with letters happens in primary school: the students are taught to form simple algebraic expressions corresponding to the data of a word problem (linear expressions with 1-2 letters). In grades 5 and 6 these skills become stronger, the students learn to read and write the basic algebraic rules, such as the properties of arithmetical operations and the properties of proportions in a letter form. A systematic study of algebra starts in grade 7, where about a half of the total time allocated to mathematics is devoted to algebraic transformations. The following skills are viewed as obligatory: to find a numerical value of an algebraic expression, to perform arithmetical operations with monomials, polynomials, and algebraic fractions, and to freely employ the formulas for $(\boldsymbol{a} \pm \boldsymbol{b})^2$, $\boldsymbol{a}^2 - \boldsymbol{b}^2$, $\boldsymbol{a}^3 \pm \boldsymbol{b}^3$, $(\boldsymbol{a} \pm \boldsymbol{b})^3$. In grade 8 this list is enlarged by adding skills of operating with square roots. Formally, this material seems to be the same for all types of curricula; however, the results achieved in the framework of different curricular differ

greatly, which is clearly seen from the texts of examination problems officially issued for general school and, say, for the curricula of M-type.

5. Other things

5.1. Regional characteristics. In Russia, the ways of teaching mathematics, as well as the methods of evaluating the results, do not vary much from region to region. Even in the absence of National Standards formally enacted, the officially approved federal curricula for general school and for specialized school leave little freedom to regional decisions. Essentially, the school chooses the ***type*** of a curriculum, the regional education approve (or reject) this choice, and after that only minor changes are possible. As was mentioned in §1, the texts of graduation exams, which are common for all within a given curriculum type, serve greatly the unification of teaching mathematics throughout the country.

5.2. Implementation strategies. Every new curriculum or new text-book intended to be wide-spread must be discussed at an especially authorized Expert Council, at the regional and then the federal level. In case of approval, some «pilot schools» are chosen for experimentation during which the texts proposed undergo relevant changes and the methods of handling the new book or curriculum are refined. If the results of such a microlocal experiment are acknowledged by the federal Expert Council to be a success, the experiment extends to all schools in an entire territorial unit or region. Only after obtaining positive results of such an extended experiment can the book or curriculum acquire the status «Recommended by the Ministry of Education», which allows it to be included in the federal list of teaching-learning materials and to be chosen for classroom work by any teacher at any school.

At present, several all-nation competitions for the authors of new mathematical school text-books are announced, with emphasis on the A- and M-profiles (in the terminology of §3).

5.3. Teacher training. Most of Russian teachers are trained in the special Pedagogical Institutes; some of these are called Pedagogical Universities. At least one such Institute can be found in every region of the

country. As a rule, pre-service teacher training lasts 5 years; the plans of study must be approved by the federal Ministry of Education. As to mathematics teachers, their training includes rather sound amount of fundamental disciplines (Algebra, Geometry, Calculus, Physics), courses in Pedagogy, Psychology, and Methods of teaching mathematics, as well as two periods of many-week practical work in schools under the supervision of their Institute's professors. The training ends with graduate exams and/or writing a sort of a graduate paper; after this, a graduate of a Pedagogical Institute is qualified as a school teacher. It is worth noting that there is no difference in teacher's training for basic school and for senior school, and any certified teacher may work in any grade from 5 to 11 (the teachers of primary school teach all subjects and are trained separately at special departments of Pedagogical Institutes). The in-service training of teachers takes place in the so-called Teacher's Improvement Institutes especially aimed at this purpose. The net of there Institutes is also wide-spread. Periodically, every teacher must take courses of in-service training.

5.4. Resources available to teachers.

a) ***Information resources***. In every school (and in every, even small, city) there is a library where a teacher can find literature for self-education. A number of publishing houses operating in the country are school-oriented. Many mathematics teachers regularly read a special weekly «Mathematics» that presents official information issued by federal educational bodies, various mathematics teaching-learning materials, discussions of new ideas, new books, etc. In every school, the mathematics teachers participate in a sort of permanently acting seminar where they share experience with one another. At the level of administrative territorial units, teachers are furnished with information at special Teacher Centers, where also optional courses in methods of teaching are often offered.

b) ***Equipment resources***. The provision of schools with equipment is very poor; the blackboard and chalk are often the only «technical tools» used by mathematics teachers. However, in the past few years computers and copying machines have appeared in many schools, and the number of

teachers for whom the use of such equipment is customary grows permanently.

5.5. Problems already detected. The social changes of the past decade have greatly affected school education in Russia. Earlier, the teaching of mathematics in basic school was aimed, largely, at the preparation of students to obtaining a sort of special education, secondary or high, but almost always technically-oriented. In connection with this, the total time allocated to mathematics was greater than that allocated to any other school subject except for Russian Language (united with Russian Literature), and in the teaching of mathematics itself emphasis was put on acquiring skills of technical nature. In the last years, the need in specialists in engineering professions has cut down sharply; instead, the demand has arisen for broadly educated people capable of working in many branches of economy. School curricula have started to include some new subjects, which borrow time from traditional disciplines such as mathematics. In these situation it seems almost impossible to reach the earlier goals, and the recognition of new objectives and new scale of values is coming slowly and painfully. Moreover, the financial resources available to schools have degraded substantially, which aggravates the content difficulties mentioned above.

5.6. Data on national/local results. Though evaluation of the results of teaching mathematics has been paid considerable attention in recent years, almost no publications of general nature that contain wide and reliable

all-nation data are available. As an exception, we mention investigations in the framework of the TIMSS program or its analogues. These studies show that the Russian students are likely to successively handle the purely curriculum topics in traditional settings; much worse do they manage problems with non-standard formulation, even those that are easy technically. Regional evaluations (e. g., in St.-Petersburg) reveal great differentiation in the results of teaching mathematics in schools with different types of curriculum.

5.7. Examples of inspiring activities. Despite the difficulties of the change-of-orientation period (see 5.5), a series of encouraging features of teaching mathematics in Russia can be indicated.

For many years in Russia there exists a close relationship between the world of schools and that of professional research mathematics. The university mathematics students and postgraduates often help schools in running optional mathematics classes; the children most interested in mathematics are frequently invited to special mathematical summer camps organized by universities. In big scientific centers, the Mathematical Societies run school sections in which teachers can meet professional mathematicians to discuss problems of teaching school mathematics.

As a more specific example, we mention that an Independent Examination Commission was set up in St.-Petersburg in 1992 by decision of the city education authorities. This Commission is aimed at elaborating new forms of examination materials and at implementing elements of external control over the examination process in schools. The work of the Commission contributes much to the creation of teacher's attitude towards new objectives of school mathematics education.

Finally, we note that the customarily well-developed system of mathematical competitions in Russia has recently been enriched by new forms that quickly gain popularity. The international game-competition «Kangaroo» is one of such new forms; unlike the traditional mathematical competitions, it attracts attention of quite a wide range of students.

6. References

For the references see the enclosed Addendum with the list of official documents and teachers materials.

7.Addendun.

List of official documents and teachers materials.

1. Official documents.

1.1. National standards of Russian schools, Part II (Mathematics and Natural Sciences), 336 pp.

1.2. Syllabi on Mathematics,- syllabus for ordinary schools;

- syllabus for schools with special stress on mathematics;

- syllabus for students with problems.

1.3. Syllabus for vocational school 28 pp.

1.4. Curriculum guide for the classes for weak students, 2 pp.

1.5. Curriculum guide for the classes with special stress on mathematics, 2 pp.

2. Assessment and evaluation.

2.1. Contents and analysis of examinations on mathematics, 144 pp.

2.2. Report on international test of level, 16 pp.

2.3. Materials on the assessment of students aged 15-17, 23 pp.

2.4. Materials for the comparative level of learning mathematics in Russia and England, 8 pp.

2.5. Diagnostic of the level on mathematics knowledge, 4 pp.

2.6. Report of the city commission mathematics examinations, 10 pp.

2.7. Results of the final attestation on mathematics, 4 pp.

2.8. Examination papers, 5 pp.

2.9. Examination papers for the grade 9, 112 pp.

2.10. Tests for the assessment of basic knowledge and skills, 34 pp.

3. Text books for students aged 14-17

3.1. Algebra 8. Y. Makarychev, 240 pp.

3.2. Algebra 8. Sh. Alimov, 240 pp.

3.3. Algebra 9. Sh. Alimov, 224 pp.

3.4. Geometry 7-9. L. Atanasyan, 336 pp.

3.5. Algebra and introduction to calculus 10-11. A. Kolmogorov, 366 pp.

3.6. Algebra and introduction to calculus 10-11. Sh. Alimov, 254 pp.

3.7. Algebra and introduction to calculus 10-11. M. Bashmakov, 384 pp.

3.8. Algebra and introduction to calculus 10-11. M. Bashmakov, 396 pp.

3.9. Algebra and introduction to calculus 10. M. Vilenkin 334 pp.

3.10. Algebra and introduction to calculus 11. M. Vilenkin, 288 pp.

3.11. Geometry 10-11. L Atanasyan, 336 pp.

3.12. Stereometry 9. A. Alexandrov, 224 pp.

3.13. Stereometry 10. A. Alexandrov, 192 pp.

4. Teachers' guidelines

4.1 Teachers' guidelines for teaching mathematics

in grade 6 (two parts), 108+102 pp.

in grade 7 (two parts), 144+70 pp.

in grade 8 (two parts), 136+40 pp.

in grade 9, 102 pp.

4.2. in grade 10-11: Functions and graphs, 96 pp.

Derivative and its application, 72 pp.

Trigonometry, 68 pp.

Exponential and logarithmic functions, 76pp.

Integral and its application, 72pp.

Equations and inequalities, 78 pp.